

Öğrenci No : .....

Adı-Soyadı : .....

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Aşağıdaki soruların cevaplarını boşluklara yazınız.

1.  $\int_1^2 (x^2 - x) dx$  integralini

a)  $n$  eşit parçaya ayırarak toplam sembolü yardımıyla hesaplayınız. (20 puan)

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(a + k \cdot \frac{b-a}{n}\right) \cdot \frac{b-a}{n}$$

0 tabanlı  $[1,2]$  aralığında  $f(x) = x^2 - x$  sürekli fonksiyonu için uygulayalım

$$\frac{b-a}{n} = \frac{1}{n}, \quad x_k = 1 + k \cdot \frac{1}{n} = 1 + \frac{k}{n}$$

$$f\left(1 + \frac{k}{n}\right) = \left(1 + \frac{k}{n}\right)^2 - \left(1 + \frac{k}{n}\right) = \frac{k^2}{n^2} + \frac{k}{n}$$

$$\int_1^2 f(x) dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{k^2}{n^2} + \frac{k}{n}\right) \cdot \frac{1}{n}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left( \frac{1}{n^2} \sum_{k=1}^n k^2 + \frac{1}{n} \sum_{k=1}^n k \right)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{1}{n^2} \cdot \frac{n(n+1)}{2}$$

$$= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^2 + n}{6n^3} + \frac{n^2 + n}{2n^2} = \lim_{n \rightarrow \infty} \frac{5n^3 + 4n^2 + n}{6n^3} = \frac{5}{6}$$

b) integral olarak hesaplayınız. (5 puan)

$$\int_1^2 (x^2 - x) dx = \left. \frac{x^3}{3} - \frac{x^2}{2} \right|_1^2$$

$$= \left(\frac{8}{3} - \frac{4}{2}\right) - \left(\frac{1}{3} - \frac{1}{2}\right)$$

$$= \frac{7}{3} - \frac{3}{2} = \frac{5}{6}$$

2.  $\int \ln(x + \sqrt{x^2 + 1}) dx = ?$

$$\ln(x + \sqrt{x^2 + 1}) = u \rightarrow \frac{1 + \frac{2x}{2\sqrt{x^2+1}}}{x + \sqrt{x^2+1}} dx = du$$

$$dx = du \rightarrow x = u$$

$$I = \int \ln(x + \sqrt{x^2 + 1}) dx = x \ln(x + \sqrt{x^2 + 1}) - \int \frac{x}{\sqrt{x^2 + 1}} dx$$

$$x^2 + 1 = t^2$$

$$2x dx = 2t dt \rightarrow x dx = t dt$$

$$I = x \ln(x + \sqrt{x^2 + 1}) - \int \frac{t dt}{t}$$

$$= x \ln(x + \sqrt{x^2 + 1}) - t + C$$

$$= x \ln(x + \sqrt{x^2 + 1}) - \sqrt{x^2 + 1} + C //$$

3.  $\int \frac{2x+3}{x^2+4x+3} dx = ?$

$$\int \frac{2x+4}{x^2+4x+3} dx - \int \frac{1}{x^2+4x+3} dx$$

$$x^2 + 4x + 3 = u$$

$$(2x+4) dx = du$$

$$\int \frac{du}{u} - \int \frac{1}{(x+2)^2 + 9} dx$$

$$x+2 = 3 \tan t$$

$$dx = 3 \sec^2 t dt$$

$$= \ln|u| - \int \frac{3 \sec^2 t dt}{9 \sec^2 t} = \ln|u| - \frac{1}{3} t + C$$

$$= \ln|x^2 + 4x + 3| - \frac{1}{3} \arctan\left(\frac{x+2}{3}\right) + C //$$



$$4. \int_1^5 \frac{2x+6}{x(x+1)^2} dx = ?$$

$$\frac{2x+6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$2x+6 = A(x+1)^2 + B(x^2+x) + Cx$$

$$= Ax^2 + 2Ax + A + Bx^2 + Bx + Cx$$

$$\boxed{A=6} \quad A+B=0 \quad 2A+B+C=2$$

$$\boxed{B=-6} \quad 12-6+C=2$$

$$\boxed{C=-4}$$

$$\int_1^5 \left( \frac{6}{x} + \frac{-6}{x+1} + \frac{-4}{(x+1)^2} \right) dx$$

$$= 6 \ln|x| - 6 \ln|x+1| + 4 \cdot \frac{+1}{(x+1)} \Big|_1^5$$

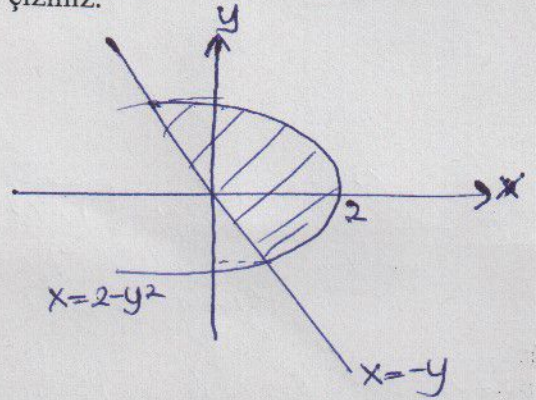
$$= 6 \ln|5| - 6 \ln|6| + \frac{4}{6} - \left( 6 \ln|1| - 6 \ln|2| + \frac{4}{2} \right)$$

$$= 6 \ln\left(\frac{5}{6}\right) + \frac{4}{6} - 6 \ln\left(\frac{1}{2}\right) - 2$$

$$= 6 \ln\left(\frac{5/6}{1/2}\right) - \frac{8}{6}$$

$$= 6 \ln\left(\frac{5}{3}\right) - \frac{4}{3}$$

5.  $x = -y$  doğrusu ile  $x = 2 - y^2$  eğrisi arasında kalan alanı hesaplayınız. Grafiğini çiziniz.



$$-y = 2 - y^2$$

$$y^2 - y - 2 = 0$$

$$\begin{matrix} -\frac{1}{2} \\ +1 \end{matrix}$$

$$y = 2, y = -1$$

$$A = \int_{-1}^2 (2 - y^2 - (-y)) dy$$

$$= \int_{-1}^2 (2 - y^2 + y) dy$$

$$= \left[ 2y - \frac{y^3}{3} + \frac{y^2}{2} \right]_{-1}^2$$

$$= 4 - \frac{8}{3} + 2 - \left( -2 + \frac{1}{3} + \frac{1}{2} \right)$$

$$= 8 - \frac{9}{3} - \frac{1}{2} = 5 - \frac{1}{2} = \frac{9}{2}$$

Sınav süresi 70 dakikadır. Başarılar.

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